New Physics Searches with Muons

Electromagnetic Interactions with Nucleons and Nuclei
Paphos, Cyprus
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Andrzej Czarnecki  🍁  University of Alberta
Outline

Three promising directions in low-energy New Physics searches:

* anomalous magnetic moments (muon vs electron)

* muonic atoms (fundamental constants and QED tests)

* lepton flavor violation (muon ---> electron)
Anomalous magnetic dipole moments
The puzzle of the muon magnetic moment

The 3.6 sigma discrepancy persists,

\[ a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 287(80) \times 10^{-11} \]

This is rather large when compared with other bounds on New Physics:

**Muon MDM**

\[ d_\mu \sim \frac{e}{2m_\mu} a_\mu^{\text{NP}} \sim 3 \cdot 10^{-22} \text{ e} \cdot \text{cm} \]

**Muon-electron transition moment**

\[ |d_{\mu \to e}| < 4 \cdot 10^{-27} \text{ e} \cdot \text{cm} \]  

**Electron EDM**

\[ |d_e| < 8.7 \cdot 10^{-29} \text{ e} \cdot \text{cm} \]
How can $g_\mu - 2$ be checked?

New experiment at Fermilab

New experimental concept at J-PARC

Can we use $g_e - 2$?

Hadronic vacuum polarization
See talk by Yangheng Zheng on Friday
New approach to $g_\mu - 2$ at J-PARC

Slower muons 300 MeV (instead of the “magic” 3.1 GeV)

Ultracold muons; no electric focusing!

Smaller ring $r = 33$ cm (instead of 7 m)

Strong, very precisely controlled magnetic field.

~ 10 times more muons than at Fermilab (compensates shorter lifetime).

<table>
<thead>
<tr>
<th></th>
<th>Brookhaven</th>
<th>Fermilab</th>
<th>J-PARC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muon momentum</td>
<td>3.09 GeV/c</td>
<td>0.3 GeV/c</td>
<td>0.3 GeV/c</td>
</tr>
<tr>
<td>gamma</td>
<td>29.3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Storage field</td>
<td>B=1.45 T</td>
<td>3.0 T</td>
<td></td>
</tr>
<tr>
<td>Focusing field</td>
<td>Electric quad</td>
<td>None</td>
<td></td>
</tr>
<tr>
<td># of detected $\mu^+$ decays</td>
<td>5.0E9</td>
<td>1.8E11</td>
<td>1.5E12</td>
</tr>
<tr>
<td># of detected $\mu^-$ decays</td>
<td>3.6E9</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Precision (stat)</td>
<td>0.46 ppm</td>
<td>0.1 ppm</td>
<td>0.1 ppm</td>
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</table>
Magnetic moment of the electron

$\alpha_e = \frac{g_e - 2}{2}$

Measured with relative error $25 \cdot 10^{-11}$

Provides the fine structure constant with the same precision,

$\alpha^{-1}(a_e) = 137.035\,999\,1736\,(331)\,(86)$

Experimental error dominates (for now)

Numerical errors from 4- and 5-loop diagrams
How to use $g_e - 2$ to check $g_\mu - 2$?

If the muon anomaly is due to New Physics, the expected effect for the electron is likely smaller by

$$\frac{m_e^2}{m_\mu^2} \sim \frac{1}{43000}$$

$$\Delta a_\mu \sim 287 \cdot 10^{-11} \rightarrow \Delta a_e \sim 7 \cdot 10^{-14}$$

This means relative uncertainty

$$\frac{\Delta a_e}{a_e} \sim 7 \cdot 10^{-11}$$

and requires a factor of 4 improvement of the latest measurement.

In addition, an independent determination of the fine structure constant is needed, with matching precision.
How to use $g_e^{-2}$ to check $g_\mu^{-2}$?

The second best determination of alpha: from atomic spectroscopy

$$R_\infty = \frac{m_e c \alpha^2}{2h}$$

Needed precision:

- $14 \cdot 10^{-11}$
- $7 \cdot 10^{-12}$ (but is it for sure?)
- $8 \cdot 10^{-11}$
- $12 \cdot 10^{-11}$
- $124 \cdot 10^{-11}$

NEW Nature 2014 Sturm et al for Rb (better for He)

improvement needed by factor ~10

$\alpha(Rb) = 1/137.035999049(90) [66 \cdot 10^{-11}]$

PRL 106, 080801 (2011)
Bound-electron $g$-2: theory needed for $u/m_e$

\[
g = 2 - \frac{2(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + \ldots
\]
\[
+ \frac{\alpha}{\pi} \left[ 1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4 (a_{41} \ln Z\alpha + a_{40}) + \ldots \right]
\]
\[
+ \left( \frac{\alpha}{\pi} \right)^2 \left[ -0.65\ldots \left( 1 + \frac{(Z\alpha)^2}{6} \right) + (Z\alpha)^4 (b_{41} \ln Z\alpha + b_{40}) + \ldots \right]
\]

\textbf{two-loop corrections}

\[
b_{41} = \frac{28}{9}
\]

\[
b_{40} = -16.4
\]

Pachucki, AC
Jentschura, Yerokhin
Another source of alpha: highly-charged ions

\[ g \simeq 2 - \frac{2 (Z\alpha)^2}{3} \]

\[ \frac{\delta\alpha}{\alpha} \sim \frac{1}{(\alpha Z)^2} \sqrt{(\delta g_{\text{exp}})^2 + (\delta g_{\text{th}})^2} \]

There is a combination of g-factors in both ions where the sensitivity to the nuclear structure largely cancels, but the sensitivity to alpha remains.

Shabaev, Glazov, Oreshkina, Volotka, Plunien, Kluge, Quint
New idea: medium-charged ions

\[ g \approx 2 - \frac{2(Z\alpha)^2}{3} \]

Hydrogen-like ion

Lithium-like ion

Combine H-like and Li-like to remove nuclear dependence; then combine with a different nucleus, to remove free-\(g\) dependence!

Much interesting theoretical work remains to be done!

Yerokhin, Berseneva, Harman, Tupitsyn, Keitel 2015
100 years ago: the g-factor!

Physics. — "Experimental proof of the existence of Ampère's molecular currents." By Prof. A. Einstein and Dr. W. J. de Haas. (Communicated by Prof. H. A. Lorentz),

(Communicated in the meeting of April 23, 1915).

When it had been discovered by Oersted that magnetic actions are exerted not only by permanent magnets, but also by electric currents, there seemed to be two entirely different ways in which a magnetic field can be produced. This conception, however, could
Muons in bound states:

muonium $\mu^+e^-$
true muonium $\mu^+\mu^-$
muonic atoms
How do we actually determine $g-2$?

Measure

$$\omega_a = \frac{g-2}{2} \frac{e}{m_\mu} B$$

$B$ from NMR

$$\omega_p = \frac{2\mu_p B}{\hbar}$$

$$\frac{e}{m_\mu}$$ from

$$\mu_\mu = g \frac{e\hbar}{4m_\mu}$$

Master formula

$$\frac{g-2}{2} = \frac{\omega_\mu/\omega_p}{\mu_\mu/\mu_p - \omega_\mu/\omega_p}$$

Measured by E821

From muonium
Muonium spectrum determines $\mu_\mu/\mu_p$

Measured to relative 120ppb (like $15 \cdot 10^{-11}$ in $a_\mu$)  
Will need improvement for the new $g$-2 results
New muonium HFS measurement in J-PARC

\[ H = -(\vec{\mu}_e + \vec{\mu}_\mu) \cdot \vec{B} + c \vec{I} \cdot \vec{J} \]

Line width ~ 100 kHz (muon decay)
Aim at ~Hz accuracy

Z boson contribution -65 Hz

Caveat: the magnetic moment in muonium differs from that of a free muon (slightly). Theory input needed!
Searches for the “true muonium”: $\mu^+\mu^-$

Arguably the most compact QED atom

Fixed-target production mechanisms:

Paradimuonium, singlet state $n^1S_0$  
Orthodimuonium, Triplet $n^3S_1$

Detection of a displaced vertex:

Part of the Heavy Photon Search (HPS)

Spectroscopic studies of $\mu^+\mu^-$ motivated by the proton radius puzzle

see talk by Aldo Antognini on Thursday
200 years ago: “protyle” hypothesis

Mass composition of air

Densities of oxygen and nitrogen, relative to air

Ammonia gives hydrogen density

Integer ratios of densities $O_2/H_2$ and $N_2/H_2$

Hypothesis: elements made of hydrogen

1815: Annals of Philosophy (anonymous!)
Lepton flavor violation
and the muon decay in orbit
Muon-electron conversion: probes various types of interactions

Non-dipole interactions are not (directly) probed by processes with external photons, by gauge invariance requirements.

New process: muon-electron conversion (as well as mu --> eee)

Variety of mechanisms:
Muon-electron conversion plans
(The Next Big Thing)

DeeMe
J-PARC
starts 2016;
aims for 1e-13 (graphite target),
followed by 1e-14 (SiC target)

COMET
Phase 1
J-PARC
7e-15

COMET
Phase 2
J-PARC
2.6e-17

Mu2e
Fermilab
2e-17
Background for the conversion search

Normal decay of the muon bound in the atom can produce high-energy electron,

\[ e \]

Spectrum has to be well understood.
Electron spectrum in a mu-decay near nucleus

Electron energy can be as large as the whole muon mass

Electron energy spectrum showing the following regions:
- **Free μ**
- **DIO**
- **TWIST 2009 shape-function region**
- **Conversion signal**
- **COMET, Mu2e high-energy region**

Diagram with axes labeled:
- $d\Gamma/dE_e$
- $E_e$
- $\frac{1}{2} m_\mu$
- $m_\mu$
Muon decay-in-orbit spectrum: the shape-function region

Experiment: TWIST
Results of TWIST (2009): less-than-perfect agreement with theory

Negative muon decay-in-orbit

- Future $\mu \rightarrow e$ conversion experiments plan to study negative muons bound to Al
- Most precise measurement ever of the muon decay-in-orbit spectrum
- Theoretical predictions include higher-order contributions from the muon+nucleus potential
- Need to include the $O(\alpha)$ radiative corrections that arise from the interaction between the muon and the outgoing electron

From Carl A. Gagliardi/TWIST
Introducing the shape function

The matrix element contains \( \delta \left( p_{e}^{2} + 2E_{e}n \cdot \pi \right) \)

Now, a trick:

\[
\delta \left( p_{e}^{2} + 2E_{e}n \cdot \pi \right) = \int d\lambda \delta \left( \lambda - n \cdot \pi \right) \delta \left( p_{e}^{2} + 2\lambda E_{e} \right)
\]

parameterizes the difference from the free decay

\[
d\Gamma = \int d\lambda S \left( \lambda \right) \ast d\Gamma_{\text{free}}
\]

The crucial advantage is that we can use the free-muon width WITH RADIATIVE CORRECTIONS (well known).
Explicit result for the shape function

\[ S(x) = \frac{8}{3\pi [1 + x^2]^3} \quad x = \frac{\lambda}{\mu Z \alpha} \]

Previously used in heavy mesons, where it cannot be computed from first principles, but can be experimentally accessed.

Mannel, Neubert, Bigi, Shifman, Uraltsev, Vainshtein
Comparison with measurement: TWIST

The spectrum is modified very significantly: effects $\sim 1/Z\alpha$

The total lifetime almost unaffected:
- phase-space reduction due to binding: compensated by the electron-nucleus attraction
- lifetime enhancement only due to time dilation $\sim (Z\alpha)^2$
Muon decay-in-orbit spectrum: the high-energy region

Experiments: Mu2e and COMET
Spectrum of the bound muon decay

It is the main background for the expected conversion signal

\[ \frac{d\Gamma}{dE_e} \sim (Z\alpha)^5 (E_{\text{max}} - E)^3 \]

105 MeV
Origin of the \((E_{\text{max}} - E)^5\) suppression

Neutrinos get no energy; The nucleus balances electron’s momentum, takes no energy.
Near the end point:

\[
\frac{d\Gamma}{dE_e} \sim |\psi(0)|^2 (Z\alpha)^2 \frac{d^3\nu_e}{\nu_e} \frac{d^3\nu_\mu}{\nu_\mu} \delta(E_{\text{max}} - E_e - \nu_e - \nu_\mu) \text{Tr} \ldots \psi_e \ldots \psi_\mu
\]

\[
\sim (Z\alpha)^5 (E_{\text{max}} - E_e)^5
\]
Radiative corrections to the electron spectrum

Expansion near the end-point

\[ \frac{m_{\mu}}{\Gamma_0} \frac{d\Gamma}{dE} = \sum_{ijk} B_{ijk} \Delta^i (\pi Z \alpha)^j \left( \frac{\alpha}{\pi} \right)^k \]

\[ \Delta = \frac{E_{\text{max}} - E}{m_{\mu}} \]

Three “small” parameters:

The expansion starts with \( B_{550} \)

The first radiative correction is \( B_{551} \)
Radiative corrections to the electron spectrum

Expansion near the end-point

Competing effects:
- vacuum polarization in the hard photon; and
- self-energy and real radiation

$$\frac{m_\mu}{\Gamma_0} \frac{d\Gamma}{dE} = \sum_{ijk} B_{ijk} \Delta^i (\pi Z \alpha)^j \left( \frac{\alpha}{\pi} \right)^k$$

$$\frac{B_{551}}{B_{550}} = \delta_H + \delta_S \ln \Delta$$

$$\delta_H = 6.31 - \frac{26}{15} \ln \frac{m_\mu}{m_e}$$

$$\delta_S = 2 \ln 2 - 2 + 2 \ln \frac{m_\mu}{m_e}$$

Szafron, AC arXiv:1505.05237
Results from the first radiative corrections

The most important correction is due to soft photons. Can be summed up to all orders (exponentiated).

\[ B_{550} + \frac{\alpha}{\pi} B_{551} \rightarrow B_{550} \left[ \Delta \frac{\alpha}{\pi} \delta_S + \frac{\alpha}{\pi} \delta_H \right] \]

\[ \delta_S = 10.1 \]

Number of electrons in the end-point bin of 1 (0.1) MeV is reduced by 11% (16%).

Remaining uncertainty in the end-point: ~ 2.5% (mainly nuclear charge distribution).
Conclusions

Many aspects of EM interactions with nuclei are crucial for precise low-energy physics:

- bound electron g-factor: vigorous experimental program in Mainz, Heidelberg and Darmstadt; gives best $m_e$, maybe alpha!

- muonic atoms: the proton radius puzzle

- muon-electron conversion: photon exchange with the nucleus modifies the decay spectrum
Efforts in LFV searches

Historical overview of selected CLFV searches:

New physics:

- (g-2)$_\mu$

Current best limit on exotic dipole moments

from Gordon Lim
Free muon lifetime

\[ \alpha^2 \text{ correction to } \Gamma (\mu(M) \rightarrow e(m) \nu \bar{\nu}) \]

Note: the blue curve is designed for \( m \sim M \), but is good even for \( m \ll M \).

So we want to exploit the expansion around \( m = M \) to get \( \alpha^3 \).
Electron propagator in an external field

\[ \frac{1}{(p_e + \pi)^2} \approx \frac{1}{p_e^2 + 2p_e \cdot \pi} \rightarrow \delta(p_e^2 + 2p_e \cdot \pi) \]

\[ \pi_\mu = i\partial_\mu - eA_\mu \]

Electron is off-shell \( p_e^2 \sim m_\mu^2 Z\alpha \)

so \( p_e \) can be written in terms of a light like \( n^2 = 0 \) vector

\[ p_e^\mu = E_e n^\mu + \delta p_e^\mu \]

\( \delta p_e \sim m_\mu Z\alpha \)

This justifies resummation \( p_e^2 \sim p \cdot \pi \)

\[ \delta (p_e^2 + 2p_e \cdot \pi) = \delta (p_e^2 + 2E_e n \cdot \pi) \]
Interpretation

Free muon decay rate, with all corrections!

\[ \frac{d\Gamma}{dE_e} = \int d\lambda \, s(\lambda) \frac{d\Gamma_{\text{free}}}{d\lambda} \left. \frac{dz}{dE_e} \right|_{z \to z(\lambda)} \]

\[ z(\lambda) = \frac{2(E_e + \lambda) + (Z\alpha)^2 m_\mu}{m_\mu + \lambda} \]

AC, M. Dowling, X. Garcia i Tormo, W. Marciano, R. Szafron