Hyperons and charmed baryons axial charges from lattice QCD

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Electromagnetic Interactions with Nucleons and Nuclei, 2015
Paphos, Cyprus, 1-7 November 2015
Introduction

Motivation

Axial charges are important quantities probing hadron structure

- low energy effective theories
- chiral perturbation descriptions
- intrinsic spin carried by the quarks

- Nucleon $g_A (1.2695(29)) \rightarrow$ benchmark calculation
- Poor results available for hyperons and charmed baryons
**Motivation**

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- low energy effective theories
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**Simulation details**

R. Baron et al. (ETMC) arXiv:1004.5284, A. Abdel-Rehim et al. (ETMC) 1507.05068

- 5 gauge ensembles produced by ETMC with $N_f = 2 + 1 + 1$ dynamical twisted mass fermions
  
  $a = 0.062$ fm, $L = 3.1$ fm and $a = 0.082$ fm, $L = 2.6$ fm

  Pion masses $\sim 210 - 430$ MeV

- 1 gauge ensemble with $N_f = 2$ at the physical pion mass by ETMC, $a = 0.093$ fm
Hyperons and charmed baryons - Spectrum

4 quark flavors \( \{ \) SU(3) subgroups \( \}\) baryons (qqq) \( \}\) of SU(4)

20plet of spin-1/2 baryons
\[
20 = 8 \oplus 6 \oplus 3 \oplus 3
\]

20plet of spin-3/2 baryons
\[
20 = 10 \oplus 6 \oplus 3 \oplus 1
\]
Calculation of Axial matrix element

Axial matrix element \( \langle B(p_f, s_f) | A_\mu | B(p_i, s_i) \rangle \)

- three-point functions, \( G(\vec{p} = \vec{0}, t_f - t_i, A_\mu(x)) \) with the axial vector current \( A_\mu(x) = \bar{q}(x)\gamma_\mu\gamma_5 q(x) \) (using the fixed current method)
- two-point functions, \( C(\vec{p} = \vec{0}, t_f - t_i) \)

Axial charge obtained directly as \( R(t_f - t_i) \xrightarrow{(t-t_i)\gg 1} g_A \)

\[
R(t_f - t_i) = \frac{G(\vec{p} = \vec{0}, t_f - t_i, A_\mu)}{C(\vec{p} = \vec{0}, t_f - t_i)}
\]

Flavor combinations:

Isovector, \( \pi \)-coupling: \( \bar{u}u - \bar{d}d \)

Isoscalar: \( \bar{u}u + \bar{d}d \)

\( \eta_8 \)-coupling: \( \bar{u}u + \bar{d}d - 2\bar{s}s \)

\( \eta_{15} \)-coupling: \( \bar{u}u + \bar{d}d + \bar{s}s - 3\bar{c}c \)
Results

$m_\pi$-dependence

Nucleon

\[ N_f = 2 + 1 + 1 \text{ Fix Sink} \]
\[ N_f = 2 \text{ Fix Sink} \]
\[ N_f = 2 + 1 + 1 \text{ Fix Curr. (this work)} \]
\[ \text{Experiment} \]

C. Alexandrou et al. arXiv:1012.0857, C. Alexandrou et al.
arXiv:1303.5979, A. Abdel-Rehim et al. (ETMC) arXiv:1507.04936

(Talk by G. Koutsou on Thursday)
$m_\pi$-dependence

**Nucleon**

\[ N_f = 2 + 1 + 1 \text{ Fix Sink} \]
\[ N_f = 2 \text{ Fix Sink} \]
\[ N_f = 2 + 1 + 1 \text{ Fix Curr. (this work)} \]
Experiment

**Σ, Ξ baryons**

\[ \text{DWF } N_f = 2 + 1 \]
\[ \text{Clover } N_f = 2 \text{ (QCDSF/UKQCD)} \]
\[ \text{Clover } N_f = 2 \text{ (CP-PACS)} \]
\[ \text{TMF } N_f = 2 + 1 + 1 \text{ (this work)} \]


(Talk by G. Koutsou on Thursday)

Results

$m_\pi$-dependence

### Decuplet

- $g_{\Lambda^{++}/g_{\sigma_{15}}}$
- $g_{\Sigma^{++}/g_{8_{15}}}$
- $g_{\Xi^{0}/g_{8_{15}}}$
- $g_{\Omega^{-}/g_{8_{15}}}$

![Graph showing $m_\pi$ dependence for Decuplet charges](image)

### Charm sector

- $g_{\Lambda_c^+/g_{8_{15}}}$
- $g_{\Sigma_{c}^{++}/g_{8_{15}}}$
- $g_{\Omega_{c}^{++}/g_{8_{15}}}$
- $g_{\Omega_{cc}^{++}/g_{15}}$

![Graph showing $m_\pi$ dependence for Charm sector charges](image)

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**C. Kallidonis (CyI)**

**Axial charges**

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Results

$m_\pi$-dependence

- Decuplet

- Charm sector

- weak pion mass dependence
Results

$m_\pi$-dependence

- Decuplet

- Charm sector

- weak pion mass dependence
- no noticeable cut-off effects

$\Lambda_{++}^0\left(g_{\pi\pi}\right)$

$\Sigma_{++}^0\left(g_{\pi\pi}\right)$

$\Omega_{++}^0\left(g_{\pi\pi}\right)$

$\Omega_{+++}^0\left(g_{\pi\pi}\right)$

$\Lambda_c^+\left(g_{\pi\pi}\right)$

$\Sigma_{++}^c\left(g_{\pi\pi}\right)$

$\Omega_{++}^c\left(g_{\pi\pi}\right)$

$\Sigma_{+++}^c\left(g_{\pi\pi}\right)$

$m_\pi^2$ (GeV$^2$)
Octet

**exact SU(3) symmetry:**

\[
\begin{align*}
g_{\pi}^N &= F + D \\
g_{\pi}^\Sigma &= 2F \\
g_{\pi}^\Xi &= F - D
\end{align*}
\]

\[
g_{\pi}^N - g_{\pi}^\Sigma + g_{\pi}^\Xi = 0
\]

**SU(3) breaking consistent with zero**

\[
\delta_{\pi}^{SU(3)} = g_{\pi}^N - g_{\pi}^\Sigma + g_{\pi}^\Xi = \sum_x c_n x^n , \quad x = \frac{m_K^2 - m_\pi^2}{4\pi^2 f_\pi^2}
\]

- breaking ~ 14% at \(x_{\text{phys}} = 0.33\)
Results
SU(3) symmetry breaking

Octet

exact SU(3) symmetry:

\[
\begin{align*}
g_N^\pi &= F + D \\
g_\Sigma^\pi &= 2F \\
g_\Xi^\pi &= F - D \\
\end{align*}
\]

\[
g_N^\pi - g_\Sigma^\pi + g_\Xi^\pi = 0
\]

\[
\delta_{SU(3)}^\pi = g_N^\pi - 2g_\Sigma^\pi + g_\Xi^\pi = \sum_x c_n x^n, \quad x = \frac{m_K^2 - m_\pi^2}{4\pi^2 f_\pi^2}
\]

破缺 $\sim 14\%$ at $x_{phys} = 0.33$

Decuplet

\[
\begin{align*}
g_\Delta^\pi &= H \\
g_{\Sigma^*}^\pi &= \frac{2}{3}H \\
g_{\Xi^*}^\pi &= \frac{1}{3}H \\
\end{align*}
\]

\[
g_\Delta^\pi - \frac{3}{2}g_{\Sigma^*}^\pi + g_{\Xi^*}^\pi &= 0 \\
g_\Delta^\pi - 3g_{\Sigma^*}^\pi - g_{\Xi^*}^\pi &= 0 \\
g_\Delta^\pi - g_{\Sigma^*}^\pi - g_{\Xi^*}^\pi &= 0
\]

破缺一致与零 $orall x$
Results
Contributions to spin

- Sum rule \( J^q = \frac{1}{2} \Delta \Sigma^q + L^q, \Delta \Sigma^q = g^q \)
- proton

![Graph showing contributions to proton spin]

\[ \frac{1}{2} \Delta \Sigma^{u+d} \]
Results
Contributions to spin

- Sum rule $J^q = \frac{1}{2} \Delta \Sigma^q + L^q$, $\Delta \Sigma^q = g^q$

- Proton

\[ \Lambda, \Delta \text{ baryons} \]

Charm baryons

- $N_f = 2$
- $N_f = 2 + 1 + 1$ (this work)
- Experiment

\[ \frac{1}{2} \Delta \Sigma^{u+d} \]

\[ \frac{1}{2} \Delta \Sigma^{u+d+s} \]

\[ \frac{1}{2} \Delta \Sigma^u \]
Results
Contributions to spin

- **Sum rule** \( J^q = \frac{1}{2} \Delta \Sigma^q + L^q, \Delta \Sigma^q = g^q \)
- **proton**

\[ \frac{1}{2} \Delta \Sigma^{u+d} \]

\[ \frac{1}{2} \Delta \Sigma^q \]

\[ N_f = 2 \]
\[ N_f = 2 + 1 + 1 \text{ (this work)} \]

- **Λ, Δ baryons**

\[ \frac{1}{2} \Delta \Sigma^{u+d+s} \]

\[ \frac{1}{2} \Delta \Sigma^u \]

\[ \frac{1}{2} \Delta \Sigma^{u+s+c} \]

\[ \frac{1}{2} \Delta \Sigma^{u+c} \]

- **Charm baryons**

\[ \frac{1}{2} \Delta \Sigma^{u+c} \]

\[ \frac{1}{2} \Delta \Sigma^{s+c} \]

\[ \frac{1}{2} \Delta \Sigma^{u+s+c} \]

\[ \frac{1}{2} \Delta \Sigma^{u+c} \]

- **weak** \( m_\pi \)-dependence
- **intrinsic spin carried by quarks** \( \sim 40 - 60\% \)

of total particle spin
Conclusions

- good agreement with existing results for the nucleon and the octet hyperons axial charges
- weak $m_\pi$-dependence of the axial charges $\rightarrow$ estimates for decuplet baryons and charmed baryons
- small $SU(3)$ breaking effects for the octet - consistent with zero for decuplet
- estimates on intrinsic spin carried by quarks for hyperons and charmed baryons

Future Work

- finalize results for $N_f = 2$ at the physical pion mass
- calculate baryon spectrum, axial charges,(etc...) for $N_f = 2 + 1 + 1$ at the physical pion mass
Conclusions

- good agreement with existing results for the nucleon and the octet hyperons axial charges
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Thank you